Delay Variability at Signalized Intersections

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Delays that individual vehicles may experience at a signalized intersection are usually subject to large variation because of the randomness of traffic arrivals and interruption caused by traffic signal controls. Although such variation may have important implications for the planning, design, and analysis of signal controls, currently no analytical model is available to quantify it. The development of an analytical model for predicting the variance of overall delay is described. The model is constructed on the basis of the delay evolution patterns under two extreme traffic conditions: highly undersaturated and highly oversaturated conditions. A discrete cycle-by-cycle simulation model is used to generate data for calibrating and validating the proposed model. The practical implications of the model are demonstrated through its use in determining optimal cycle times with respect to delay variability and in assessing level of service according to the percentiles of overall delay.

The ability to accurately quantify vehicle delays at signalized intersections is a critical component for the planning, design, and analysis of signal controls. As a result of random fluctuations in traffic flow and interruptions caused by traffic controls, delays that individual vehicles experience at a signalized approach are often subject to highly stochastic and time-dependent variation. It has been increasingly recognized that the estimate of the variability of delays is also of importance for many applications (*1–3*). For example, knowledge of the variability of delays makes it possible to estimate the confidence limits about the mean delays and thus provide a more informative comparison of alternative signal plans in identifying optimal signal settings. By considering the variability of delay, more reliable signal control strategies may be generated, potentially leading to improved levels of service at signalized intersections.

The problems of estimating delays at signalized intersections have been extensively studied in the literature; however, the majority of the work has focused on developing models for estimating mean delay—a point estimate of stochastic delays. Detailed discussions of these average delay prediction models have been provided by Allsop (*4*), Newell (*5*), and Hurdle (*6*). However, much less work has been done to quantify the variability of delay at a signalized approach. Teply and Evans (*7*) analyzed the delay distribution at a signalized approach for the evaluation of signal progression quality. They observed that most of the delay distributions are bimodal, and a point estimator is not adequate to describe these distributions. By considering the cyclic overflow delay over time as a Markov chain, Kimber and Hollis (*8*), Cronje (*9*), and recently Olszewski (*1*, *10*) developed numerical methods to calculate the average delay and time-dependent distribution of average cyclic delay. This type of model, although capable of completely specifying the delay distribution, requires substantial computational resources for calculating and storing state and transition probabilities and therefore is not well suited for use in practical situations. The objective of this paper is to develop an analytical model for the estimation of the variability of delays at signalized intersections with a specific focus on predicting the variance of delays of vehicles traversing a signalized approach during a given time interval.

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An approximate model for predicting the variance of delays is presented. The methodologies applied to develop the approximate model are outlined, followed by presentation of the development of the approximate model. Then the discrete cycle-by-cycle simulation model that was developed for calibrating and validating the proposed model is described. This simulation model is then used to generate data for calibrating and validating the proposed model under a variety of signal operating conditions. Applications of the developed model are demonstrated last through its use in a sensitivity analysis and in determining reliability-oriented optimal cycle times and levels of service. Finally, conclusions and recommendations are presented.

ASSUMPTIONS AND NOTATION

The delay that a particular vehicle experiences when it travels through the approach to a signalized intersection depends on a number of factors, including the probabilistic distribution of arrival flow, signal timings, and the time when the vehicle arrives at the approach. In a real application environment, many of these factors are random variables, which makes accurate estimation of this delay a very complicated process. As an initial research effort, the following idealized road traffic and signal control conditions are considered in this paper:

1. The intersection approach consists of a single through lane controlled by a fixed-time signal. The approach has unlimited space for queuing and has a constant saturation flow rate.
2. The vehicle arrival at the approach is a random variable with a known probabilistic distribution. The rate of vehicle arrivals during the evaluation time is assumed to be constant. No initial queue is present at the beginning of the evaluation time. The flow rate increases abruptly from zero to the rate for the evaluation time. The traffic stream consists only of passenger-car units (pcu).

Consider the cumulative arrival and departure of vehicles during the time interval $[0, T]$ at the stopline of a signalized approach as illustrated in Figure 1. The delay for a particular vehicle arriving at time t, called *overall delay* and noted as D, is considered to include two components: uniform delay and overflow delay, as follows:

$$D = D_1 + D_2 \tag{1}$$

where the uniform delay component, D_1, is defined as that portion of delay incurred by a vehicle when the approach is undersaturated and all vehicles arrive uniformly. The overflow delay component, D_2, represents that portion of delay caused by temporary overflow queues resulting from the random nature of arrivals and by continuous

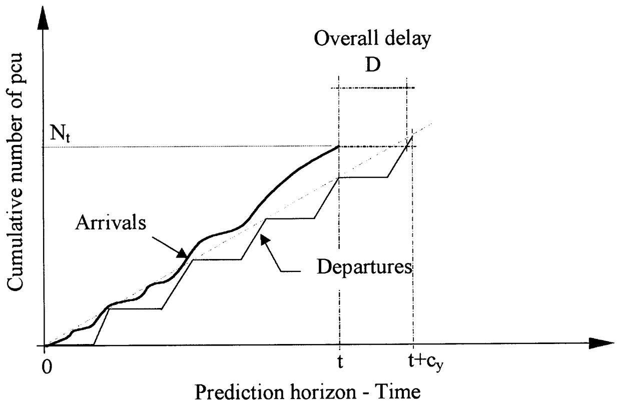

FIGURE 1 Queuing diagram illustrating components of delay.

overflow when the arrival rate during the time period [0, T] exceeds the capacity.

The estimation of the overflow delay component in Equation 1 is complicated as a result of the complex time-dependent stochastic nature of the queuing process, and currently no theory is available for the development of a single analytical model suitable across all saturation levels. Past research has mainly focused on developing approximate models for estimating the average overall delay using simulation data as a mechanism to obtain data for calibration (*8*, *11–15*). A number of similar delay models are available to provide estimates of this measure. For example, the Canadian delay model uses Equation 2 to estimate the average overall delay (*13*, *16*), in which the units of some parameters have been changed for use in this paper:

$$E[D] = k_f \frac{c_y(1-\lambda)^2}{2(1-\lambda x_1)} + 0.25T\left[(x-1) + \sqrt{(x-1)^2 + \frac{4x}{c_a T}}\right] \quad (2)$$

where

$E[D]$ = average overall delay (s);
T = evaluation time (s);
c_y = cycle time (s);
$\lambda = g_e/c_y$;
g_e = effective green interval duration (s);
k_f = adjustment factor for effect of quality of progression, defined as $k_f = (1-p)f_p/(1-\lambda)$ (p is the proportion of vehicles arriving during the green interval, and f_p is a supplemental adjustment factor for platoon arrival type; note that this study does not consider the effect of signal progression; i.e., $k_f = 1.0$);
c_a = capacity (pcu/s), determined by $s\lambda$, where s is saturation flow rate (pcu/s);
x = degree of saturation, defined as q/c_a;
q = average arrival flow rate from time 0 to time T (pcu/s); and
x_1 = minimum of (1.0, x).

The development of a model for the variance of overall delay, Var[D], is considered, which is defined as the summation of the variance of uniform delay and the variance of random delay:

$$\text{Var}[D] = \text{Var}[D_1] + \text{Var}[D_2] \quad (3)$$

where Var[D_1] is the variance of uniform delay, defined as the variance of delay that would be experienced by vehicles when all vehicles arrive uniformly at a constant arrival rate less than or equal to capacity; Var[D_2] is the variance of random delay or the difference between the variance of overall delay and the variance of uniform delay. The variance of uniform delay can be derived theoretically and the variance of overflow delay can be directly calibrated from simulation data. A detailed description of the development of these models is provided next.

APPROXIMATE MODEL FOR VARIANCE OF OVERALL DELAY

The variance of uniform delay, Var[D_1], represents the variation of uniform delay that would be experienced by vehicles arriving during time interval [0, T]. This variation results from the uncertainty of the vehicle's arrival time during each cycle of the interval. The vehicle can arrive at any moment within a cycle and thus experience variable delays as a result of the signal control. An estimate of this variance component can be obtained theoretically on the basis of a deterministic queuing model with vehicles arriving uniformly during the cycle (*2*):

$$\text{Var}[D_1] = \frac{c_y^2 \cdot (1-\lambda)^3 \cdot (1 + 3\lambda - 4\lambda x_1)}{12(1-\lambda x_1)^2} \quad (4)$$

In order to establish a model for the variance of delay caused by an overflow queue, two extreme traffic conditions are first investigated: undersaturated conditions ($x < 1.0$) and oversaturated conditions ($x > 1.0$). For undersaturated conditions, overflow delay experienced by a vehicle arriving during the time interval [0, T] is mainly caused by occasional overflows of traffic from each cycle. The relationship between the variance of this delay and the degree of saturation can be approximated from the well-known Pollaczek-Khintchine formula for an M/G/1 system (for the general formula and derivation) (*17*) by supposing that the signal is acting as a server with a constant service time $1/c_a$, as follows:

$$\text{Var}[D_2] \approx \frac{x \cdot (4-x)}{12 c_a^2 \cdot (1-x)^2} \quad (5)$$

It should be emphasized that the foregoing model is merely an approximate estimate of the variance because a steady state may not be reachable during time interval [0, T]. Nevertheless, the equation can be used to illustrate the qualitative relationship between the variance of delay and the degree of saturation. With this assumption, the variance is time-independent and an infinite variance would be predicted as the degree of saturation (x) approaches unity. In reality, at high degrees of saturation, the system is not likely to settle into a steady state by time T. Consequently, it can be expected that Equation 5 provides a reasonable approximation of the variance only under light traffic conditions ($x \ll 1.0$).

If the intersection approach is highly oversaturated during time period [0, T], there is a high probability that an overflow queue always exists during the period from time 0 to time T. Consider a vehicle arriving at time t during time period [0, T]. The overflow queue for a vehicle arriving at time t, Q_t, can be determined as the total arrivals minus the total departures:

$$Q_t = N_t - c_a \cdot t \quad (6)$$

The number of arrivals, N_t, is a random variable with a mean equal to qt. The delay experienced by the vehicle can then be simply determined on the basis of the overflow queue:

$$D_2 = \frac{N_t - c_a \cdot t}{c_a} \tag{7}$$

On the basis of Equation 7, the variance of delay for vehicles arriving during time interval [0, T] can be obtained by assuming that the arrival time t is a random variable with known distribution:

$$\begin{aligned}\mathrm{Var}[D_2] &= \frac{E[\mathrm{Var}[N_t - c_a t|t]] + \mathrm{Var}[E[N_t - c_a t|t]]}{c_a^2} \\ &= \frac{E[\mathrm{Var}[N_t|t] - 0] + \mathrm{Var}[E[N_t|t] - c_a t]}{c_a^2} \\ &= \frac{E[\mathrm{Var}[N_t|t]] + \mathrm{Var}[qt - c_a t]}{c_a^2} \\ &= \frac{E[\mathrm{Var}[N_t|t]] + (q - c_a)^2 \mathrm{Var}[t]}{c_a^2}\end{aligned} \tag{8}$$

If the ratio of the variance to the mean of the vehicle arrivals, denoted as I_a, is assumed to be constant during the time interval [0, T] and given, then

$$\mathrm{Var}[N_t|t] = I_a E[N_t|t] = I_a q_t T \tag{9}$$

Note that if the vehicle arrivals follow a Poisson distribution, I_a is equal to 1. In this study, Poisson arrival is assumed, but the parameter I_a is still used for the convenience of future extension. With Equation 9, Equation 8 can be further expressed as

$$\mathrm{Var}[D_2] = \frac{I_a q E[t] + (q - c_a)^2 \mathrm{Var}[t]}{c_a^2} \tag{10}$$

If it is assumed that the arrival time is uniformly distributed during the time interval [0, T], Equation 10 can be further expressed as

$$\mathrm{Var}[D_2] = \frac{I_a T x}{2c_a} + \frac{T^2(1 - x)^2}{12} \tag{11}$$

It must be emphasized that Equation 11 is valid only when an overflow queue is present during the period from time 0 to time t. In reality, however, it is possible that no overflow queue exists at time t, and consequently no overflow delay is experienced. Therefore, it can be concluded that Equation 11 represents an upper-bound estimate of the variance of overflow delay. The actual variance would be lower than that predicted by Equation 11, but the prediction error should become smaller as the degree of saturation increases and the associated likelihood of overflow queuing increases.

Figure 2 shows the relationships between the variances of overflow delay as functions of the degree of saturation represented by Equations 5 and 11. Both curves are only appropriate within certain flow domains: either highly undersaturated or highly oversaturated traffic conditions. Consequently, it is hypothesized that the true relationship between the variance and the degree of saturation follows the dashed curve in Figure 2. It can be observed that it is difficult, if not impossible, to derive the functional relationship for the transitional curve directly from Equations 5 and 11 through the traditional coordinate transformation technique. Therefore, the nonlinear function, expressed in Equation 12, is proposed to model the variance:

$$\mathrm{Var}[D_2(t)] = \left\{\frac{I_a T x}{2c_a} + \frac{T^2(1 - x_1)^2}{12}\right\} e^{-\left(\frac{x_0}{x}\right)^\beta} \tag{12}$$

where $x_1 = \max\{1, x\}$.

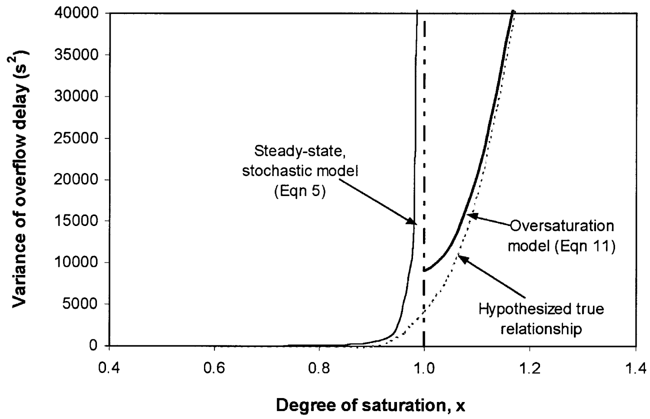

FIGURE 2 Models for variance of overflow delay.

The parameters x_0 and β determine the shape of the delay curve, and their values need to be calibrated. It can be observed that the proposed function has two desired attributes. First, the function is asymptotic to the model for oversaturated conditions (Equation 11). Second, similar to the undersaturated model (Equation 5), the function goes to zero as x approaches zero. However, although these characteristics are necessary, they do not of themselves demonstrate that the proposed function is realistic. Therefore, data from a simulation model were used to calibrate appropriate values for x_0 and β and to validate the calibrated model, as discussed in the next section.

Expressions for the variances of uniform delay and overflow delay having been developed, the variance associated with the overall delay (Equation 7) can be expressed as

$$\mathrm{Var}[D(t)] = \frac{c_y^2 \cdot (1 - \lambda)^3 \cdot (1 + 3\lambda - 4\lambda x_1)}{12(1 - \lambda x_1)} + \left\{\frac{I_a T x}{2c_a} + \frac{T^2(1 - x_1)^2}{12}\right\} e^{-\left(\frac{x_0}{x}\right)^\beta} \tag{13}$$

SIMULATION MODEL

In order to obtain data to calibrate and validate the proposed models, a discrete cycle-by-cycle simulation system was developed.

Logic

The simulation model explicitly models the delay that a vehicle experiences when it traverses a signalized intersection approach. The approach is used exclusively for through traffic and is controlled by a pretimed traffic signal. The vehicle arrivals are randomly distributed with the vehicle headway following a negative exponential distribution with a minimum headway equal to 1 s.

The vehicle discharge pattern during the green interval depends on the queue status at the approach. If no queue is present when a vehicle arrives, it can immediately be discharged without delay. Otherwise, the vehicle must wait until the discharge of the queued vehicles ahead of it. Vehicle discharge headway is determined on the basis of saturation flow rate.

The simulation starts with no queue present and resets the queue size to zero whenever the elapsed clock time reaches a prespecified evaluation time. The simulation terminates once the required total number of cycles has been simulated. The arrival time and delay associated with each vehicle are recorded for use in the analysis stage. Information such as the mean and variance of delays experienced by vehicles arriving during the evaluation time can then be derived.

Verification

Before the simulation model was used to generate data for calibrating and testing the proposed models, it was verified against results from other available models. Two comparisons were made. First, the average overall delays obtained from the simulation model for a given evaluation period under different saturation ratios were compared with the results from the Australian (*12*), Canadian (*13*), *Highway Capacity Manual* (HCM) (*18*), and Markov chain models (*1*). For convenience, the scenario used in this comparison is the same as that used by Olszewski (*1*) for a similar purpose. The evaluation period duration was 15 min. The signal timing consisted of a cycle time of 60 s, an effective green interval of 24 s, and a saturation flow of 1,800 pcu/h. A total of 6,000 cycles, corresponding to 100 h of traffic flow, were simulated for each degree of saturation. It was estimated that this number of simulations would result in an estimation error of less than 0.5 s at a significance level of 95 percent.

Figure 3 illustrates the average overall delay obtained from the simulation model and the four other methods. It should be noted that the overall delays associated with the HCM model were obtained by multiplying the stopped delays from the HCM formula by 1.3 to convert stopped delay to overall delay. The Markov chain model assumes Poisson arrivals and constant departure during the green interval. As would be expected, the simulation results are almost identical to those of the Markov chain model. Among the three other models, the Australian model shows the best agreement with the simulation model under all levels of saturation, and the Canadian model provides the best agreement with the simulation for oversaturated conditions. It should be noted that the differences among the HCM, Canadian, and Australian delay equations were expected and have been addressed by Akcelik (*19*).

The objective of the second comparison was to provide an indication of the validity of the simulation model in estimating the variance of delays. The simulation results were compared with those reported by Olszewski (*1*) in which the exact means and variances of delays under various levels of saturation were obtained for a given case from a Markov chain model. The system parameters are the same as those for the previous comparison except that the evaluation time was 30 min instead of 15 min. In this comparison, the number of cycles to be simulated was estimated on the basis of an analysis of the confidence interval for the variance. It was estimated that a total of 6,000 cycles for each degree of saturation would yield an estimation error for the standard deviation of less than 2 s at a significance level of 95 percent. Figure 4 shows the standard deviations of delay estimated by the simulation model and those provided by Olszewski (*1*) from the Markov chain model. It can be observed that the estimates of the standard deviation of delay from the simulation model are quite consistent with those obtained from the Markov chain model. The overestimation of the standard deviation of delay by the simulation model, especially in the range $x < 1.0$, was expected because the Markov chain model does not consider the variation of travel time within the cycle as quantified by Equation 4.

MODEL CALIBRATION AND VERIFICATION

Calibration

To determine the appropriate parameter values for the overflow delay variance model shown in Equation 3, a two-step sequential calibration procedure was performed. The first step is to find the x_0- and β-values that would produce the best fit between the estimates of the variance of overflow delay from Equation 12 and the estimates from the simulation model (representing the true values) for a given cycle time (c), effective green interval (g_e), and evaluation time (T). Following the definition of Equation 3, the variance of random delay from the simulation is obtained as the difference between the variance of overall delay calculated from the delay of simulated vehicles and the variance of uniform delay from Equation 4.

Because of the nonlinear relationship between the variables, a nonlinear regression process was conducted by first transforming Equation 12 into an equivalent linear equation:

$$Y = a + bX \tag{14}$$

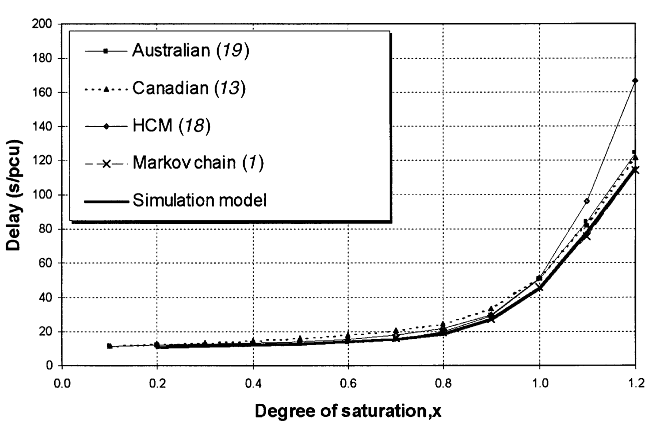

FIGURE 3 Average overall delay estimated by Australian, Canadian, HCM, Markov chain, and simulation models (c_y = 60 s, g_e = 24 s, s = 1,800 pcu/h, and $t = t_e$ = 15 min; simulated cycles = 6,000).

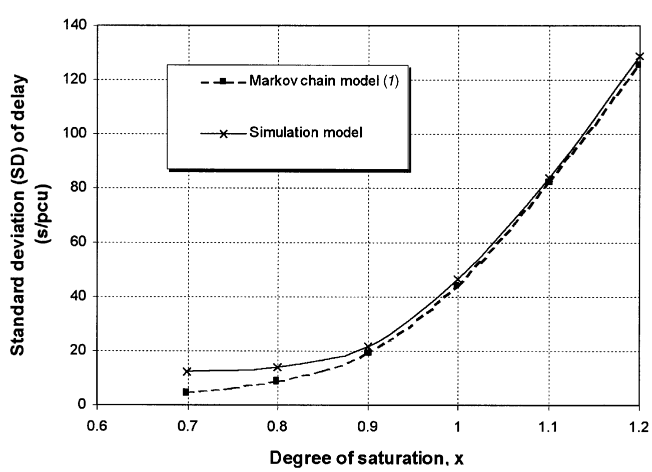

FIGURE 4 Standard deviation of delay estimated by Markov chain model and simulation model (c_y = 60 s, g_e = 24 s, s = 1,800 pcu/h, and $t = t_e$ = 30 min; simulated cycles = 6,000).

where

$$Y = \ln\left\{\ln\left(\left[\frac{I_a q x}{c_a} + \delta\frac{T^2(1-x)^2}{12}\right]\right) - \ln(\mathrm{Var}[D_2])\right\}$$

$X = \ln(x)$,
$a = \beta \ln(x_0)$, and
$b = -\beta$.

The simulation model was used to obtain the values of the variance of overflow delay ($\mathrm{Var}[D_2]$), which is the difference between the variance of overall delay and the variance of uniform delay calculated from Equation 4, under various combinations of c_a, x, and T. These data were transformed to X- and Y-values as in Equation 14. For a set of prespecified x-values, linear regressions were performed to determine the values of a and b, which were subsequently transformed back to values for x_0 and β. The data points used in regression were determined by simulation by fixing the values of x, g_e, and T and varying the degree of saturation x from 0.8 to 1.2 with an increment of 0.05. Each data point results from a simulation of 15,000 cycles. The regressed x_0 and β, together with (c_y, g_e, T), form a new data point (c_y, g_e, t, x_0, β). By changing the values of the parameter set (c_y, g_e, T) and repeating the regression analysis, a number of such data points can be obtained. In this study, a total of 18 points were generated with the following combinations of parameters: $c_y = \{70, 90, 120\}$; $\lambda = g_e/c_y = \{0.2, 0.5, 0.8\}$; $T = \{900, 3600\}$. It was found that the linear relationship shown in Equation 14 is statistically significant for each of the 18 combinations with a minimum R^2 of 0.95, which indicates that the proposed functional form is appropriate.

In the second step, a series of correlation analyses of the relationships between the parameters (x_0, β) and (c_y, g_e, T, λ, T/c_a) were conducted and the following best-fit equations were obtained:

$$x_0 = 0.947 + 1.330 * 10^{-6}\, T/c_a + 0.157\lambda \tag{15}$$

($R^2 = 0.87$, $t_1 = 13.07$, $t_2 = 3.84$)

$$\beta = 8.294 + 6.080 * 10^{-4}\, T/c_a \tag{16}$$

($R^2 = 0.93$, $t_1 = 13.07$)

The obtained high R^2-values indicate that both equations explain a large portion of the variation in the simulated data. All t-values are greater than the critical t-value at the 5 percent level of significance, which indicates that the included parameters are statistically significant.

Evaluation

The simulation system is first used to estimate the variance of overall delay corresponding to various evaluation times and traffic conditions. A total of 210 combinations were simulated with the following combinations of parameters: $c_y = \{50, 60, 80, 100, 120\}$, $\lambda = \{0.3, 0.5, 0.7)$, $t = \{900, 3600\}$, and $x = \{0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2\}$.

Figure 5 shows the correlation between the standard deviation of the delay obtained by the analytical model and the simulation results. Each point represents the result of simulation runs of 15,000 cycles. The approximate model exhibits no apparent bias and has a high correlation with the simulated estimates ($R^2 = 96.1$ percent).

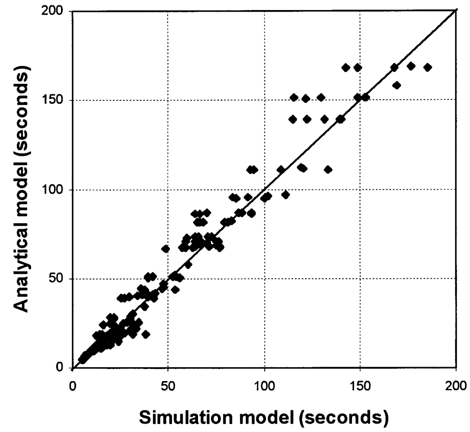

FIGURE 5 Correlation of standard deviations of overall delay estimated by analytical model with simulation results (s = 1,800 pcu/h; simulated cycles = 15,000 per combination).

The calibrated model was further evaluated using results from Olszewski (*1*) in which the exact variances of overflow delays under various levels of saturation were obtained for a given case from a Markov chain model. The values of the system parameters for the case as well as the results are shown in Figure 6. It can be observed that the estimates of the standard deviation of delay from the simulation model are very consistent with those obtained from the Markov chain model.

APPLICATIONS OF VARIANCE MODEL

Optimal Cycle Time

Average overall delay has traditionally been used as one criterion in determining optimal cycle times. An examination is made to see if there is an optimal cycle time that minimizes the variability of delay that individual vehicles experience at a signal-controlled intersection. An idealized two-phase, four-approach intersection with equal flows

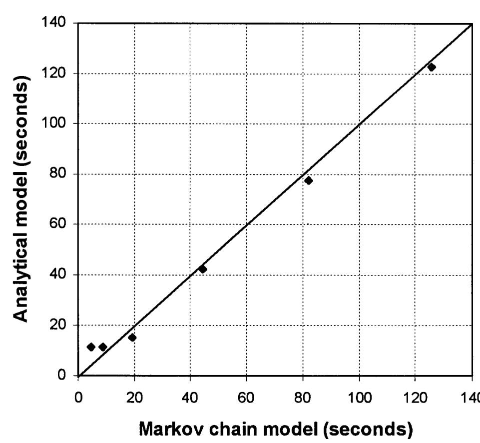

FIGURE 6 Standard deviation of overflow delay estimated by Markov chain model (*1*) and simulation model (c_y = 60 s, g_e = 24 s, s = 1,800 pcu/h, and T = 30 min).

on all approaches is considered. The saturation flow rate is 1,800 pcu/h and the lost time at each phase is 4 s. Figure 7 shows the relationship between the variance of overall delay and the average overall delay as a function of cycle time. It can be observed that the average overall delay and the variance of overall delay have similar trends with respect to cycle length. Furthermore, the range of optimal cycle times with respect to average overall delay (q = 600 pcu/h: optimal c_y = 40 ~ 60 s, and q = 800 pcu/h: optimal c_y = 80 ~ 100 s) overlaps with those determined on the basis of minimizing the variance of overall delay (q = 800 pcu/h: optimal c_y = ~40 s and q = 800 pcu/h: optimal c_y = 90 ~ 120 s). This finding indicates that for the scenarios examined, the current practice of determining optimal cycle lengths on the basis of minimizing average overall delay is appropriate with respect to the objective of minimizing the variance in overall delays.

Variability of Level of Service

The possible use of delay variability in quantifying level of service for signalized intersections is illustrated in this section. In the HCM (*18*), level of service for signalized intersections is defined in terms of average overall stopped delay. With the ability to estimate the variance of overall delay, it is feasible to integrate the concept of reliability into design and analysis of signalized intersections. For example, delay of a certain percentile, instead of average value, can be used to define the level of service. A 95th-percentile delay means that 95 percent of the vehicles would experience delay less than or equal to this delay. The percentile value can be approximately estimated using $E[D] + z_\alpha (\text{Var}[D])^{1/2}$, where z_α is a statistic for the normal distribution and can be determined on the basis of the prespecified reliability level. Figure 8 shows average overall delay and 90th-percentile delay (with $z_\alpha \approx 1.3$) under different degrees of saturation. It is assumed that ranges of delay values used in defining each level of service in the HCM are also applicable to individual vehicles, as shown in Figure 8. It can be observed that for the given case with a degree of saturation of 0.9, the average overall delay is 20 s, which would yield level-of-service (LOS) C (point *a*). However, if the 90th-percentile delay is used, the level of service would be D (point *b*). On the other hand, in order to guarantee that 90 percent of the vehicles going though the intersection approach experience LOS C or higher, the degree of saturation needs to be reduced to 0.8 (point *c*) by either increasing the capacity or decreasing the demand.

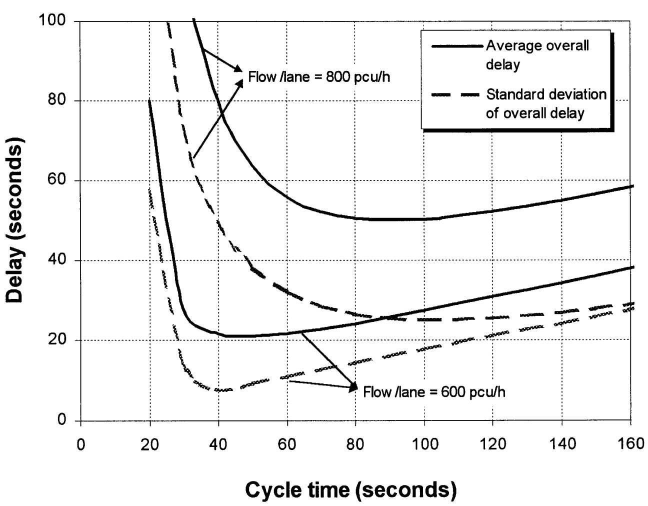

FIGURE 7 Relationship between optimal cycle times with respect to mean and standard deviation of overall delay (two-phase, four-approach intersection with equal flows on all approaches; saturation flow = 1,800 pcu/h; lost time = 4 s/phase).

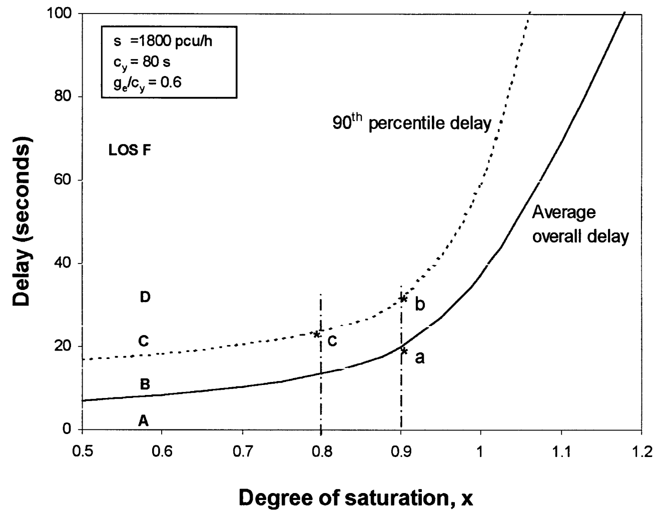

FIGURE 8 Level of service and delay variability.

CONCLUSIONS AND FUTURE RESEARCH

The development of an analytical model for estimating the variance of delay at signal-controlled approaches is described. The model was constructed on the basis of the delay evolution patterns under two extreme traffic conditions: highly undersaturated and highly oversaturated. A discrete cycle-by-cycle simulation model was developed and used to generate data for calibrating and validating the proposed models. The results of a correlation analysis indicate a remarkable agreement between the model estimates of the standard deviation of delay and simulation results (R^2 = 96.1 percent).

The developed model provides a valuable tool for the planning, design, and analysis of signal controls. Practical applications have been demonstrated through its use in determining optimal cycle times with respect to delay variability and in assessing level of service according to the percentiles of overall delay.

The proposed analytical models were calibrated and validated with simulation results that are based on several important assumptions, including random traffic arrivals with constant flow rate and unlimited queuing space. These assumptions may be overly restrictive and are likely to be violated in practice. The impact of these assumptions on the validity of these models has not yet been determined. It is recommended that future research focus on the following aspects: the potential impacts of the assumptions applied in this paper should be quantified, and field data should be used in conjunction with simulation results to calibrate and verify the proposed models.

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